THE OPTIMIZATION OF THE ALLOCATION OF PERSONNEL EFFORT TO MINE WARFARE

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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE

United States Naval Postgraduate School
Monterey, California

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FOREWORD

The development presented here was suggested for investigation by Professor C. C. Torrance. Further acknowledgement must be made to Professor Torrance and to Professor B. J. Lockhart for their invaluable assistance in completing the work.

ABSTRACT

The investigation undertaken here is to determine if there can be found an optimum allocation of personnel effort to mine countermeasures, and to determine the effects of a failure to locate or to comply with this allocation if it exists. The solution proceeds with the application of the formulas of probability as related to mine warfare and with the use of Lanchester's Square Law for calculating casualties to personnel in a combat situation.

The graphs presented in Figs. 1 and 2 demonstrate the optimum allocation of personnel, obtained for each of several values of a numerical parameter of effectiveness of mine combatting gear assumed to be used. From these graphs may also be inferred the practical desirability of developing an equipment with the highest possible value of this parameter.

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1. Introduction.

The problem of determining the size of the force needed to combat an enemy mine capability has never been solved. It may be attacked in several ways. In this article an attempt is made to show that for a given set of conditions, an optimum allocation of personnel can be obtained. The conditions imposed greatly simplify the problem, but it is believed that the solution obtained is indicative of the solution to the general case.

TABLE OF SYMBOLS

N- the number of mines originally placed in channel

U- the number of mines remaining in the channel at any later time

s- the number of minesweepers employed

h- the number of minehunters employed

p_c- the probability that a ship is sunk while attempting to pass through the minefield

p_r- the probability that a mine combatting unit effectively eliminates a mine that is in the minefield

S- speed over the ground; used for minehunts only

C_m- the number of casualties to personnel due to mines

L- the total number of casualties to personnel due to mines and combat

m- the number of personnel remaining in the Red force at any time after the start of the combat

m'- the number of personnel with which the Red force starts the combat

n- the number of personnel remaining in Blue force at any time after the start of the combat; in particular, the number remaining after 1000 casualties have been inflicted on the Red force

n'- the number of personnel in Blue force that are designated land combat troops rather than mine combat units

n_o- the number of the Blue force land combat troops that proceed safely through the minefield; equals $n'-C_m$ and is the number with which the Blue force starts the combat

STATEMENT OF ASSUMPTIONS

A combat situation exists wherein equal Red and Blue forces lie opposed to each other on opposite sides of a minefield that has been laid by the Red force. The Blue force is required to pass through the minefield and to attack the Red force. Each mine combatting unit employed by the Blue force consists of 50 men who are then ineligible for ground combat.

The remainder of the Blue force is sent through the minefield. The casualties thus incurred are determined from the equations developed by Dr. R. K. Reber. However, for sweeping, rather than the standard swept path, it has been assumed that the channel dimensions and the characteristics of the mine combatting equipment are such, that, on any given pass, each unit will eliminate a certain percentage of the mines in the channel. This makes the number of mines remaining in the channel a simple geometric progression. In addition, each unit is restricted to one pass due to a time limitation imposed by combat conditions.

The firing width is taken to be the "sinking damage" width for all ships so that all mine explosions are ship sinkings.

METHOD OF PROCEDURE

Each side is assigned a number of men. From the number of attackers a mine combatting force is assigned. For a given number of mines initially in the channel, the number remaining after the completion of the efforts of this force is computed as in Appendix A. The remainder of the Blue force is sent through the minefield. The number of casualties due to mines, a function of the number of mines remaining, is then computed, and subtracted from the number of men entering the minefield to yield the number remaining in the Blue force at the start of combat.

Lanchester's Square Law is applied to the Red and Blue forces at this point, and, for this report, as in Appendix B, the number of casualties to Blue forces that are incurred while inflicting 1000 casualties to the Red force is computed.

The total casualties, from mines and from combat, are to be minimized. Thus for each of the various mine combatting numerical parameters, the number of total casualties for each size of mine combatting force is computed. The minimum of this number of casualties is obtained at the optimum distribution of personnel effort between mine countermeasures and ground combat.

INTERPRETATION OF RESULTS

The graphs in Figs. 1 and 2 show the results of the calculations as made in the Appendix. In Fig. 1 is plotted, for the case of minesweeping only, the variation of total casualties with number of sweepers. Four values of p_r are shown. In each case a minimum casualty point or optimum distribution is obtained. The curves are seen to be steeper for personnel distributions with fewer than optimum mine combatting units than for distributions slightly greater than optimum. The implication is that for this type of situation an error on the side of over-emphasis is less expensive in terms of personnel casualties than one of under-emphasis.

A second observation can be made about the data. The casualties incurred in accomplishing a given mission are greatly reduced if the effectiveness of the mine combatting gear is increased. This means that low effectiveness of gear cannot be compensated for by increased effort, under the combat conditions assumed.

The data used in Fig. 2 were obtained for the case of minehunting only and are seen to conform in general to the observations made above. The effects of combining the two in various ratios has not been studied here, but it is believed that results would be obtained that are similar to those found here.

CONCLUSIONS AND RECOMMENDATIONS

For this type of combat it is seen that a level of effort exists, for each value of effectiveness of mine combatting gear, that will minimize the total casualties to personnel. For the case of $p_r = 0.5$ this level is 13% of the total effort.

Therefore it is recommended that an investigation be made to determine the effectiveness of present types of gear in the sense of this report. This would establish the required level of effort, and would be an excellent criterion for determining the force requirement for mine warfare.

APPENDIX

CASUALTIES DUE TO MINES

A. Countermeasures include minesweeping only.

For the assumption that the number of mines eliminated by each sweeping type unit on one pass is a constant percentage of the mines present in the channel, the number of mines, remaining in the channel after "s" passes by sweepers is given by the formula.

(1) $U = N(1-p_r)^s$

Where p_r is the percentage of the number of mines present that are eliminated on one pass, and N is the number of mines originally present.

An expression for computing the probability that a ship explodes a mine while transiting the minefield has been developed by Dr. R. K. Reber.

and is reported in Ref. (a) as

(2) $P_s = 1-(1-W/A)^U$

Where W/A is the ratio of ship firing width to channel width. Since W/A has been assumed to be 1/3 and all mine explosions are assumed to cause sinking damage, the formula may be restated as

(3) $P_c = 1-(2/3)^U$

Where P_c is now the probability that a ship is sunk in transiting the field. For the case of identical ships, each carrying the same number of men, P_c can also be used as the expected percentage of personnel casualties due to mines.

Using these formulas the date, for personnel casualties due to mines, listed in Tables 1, 2, 3 and 4 are calculated. For example, in Table 1,

$$P_r = 0.2$$

Let $s = 30$

Then, for $N = 100$,

$$U = (100)(.8)^{30} = 0.129$$

and,

$$P_c = 1-(2/3)^{0.129} = 0.051$$

Now,

$$n' = 5000 - 50A = 3500$$

and,

$$C_m = n'P_c = (3500)(0.051) = 178$$

Finally,

$$n_o = n' - C_m = 3500-178 = 3322$$

B. Countermeasures include minehunting only.

For the case of minehunting the expression for U is given in Ref. (a) as

$$(4) \quad U = Ne^{-\bar{m}_o \gamma}$$

where γ is a parameter taken as 2.56, and $\bar{m}_o$ is a measure of the effective area of the channel covered. An arbitrary set of characteristics is then selected to represent $\bar{m}_o$, such as

6 hrs. working time

30 mile length of channel

$P_r = .9$

$S = .45$ KTS

$h =$ number of hunters

Formula (4) then becomes

(5) $U = Ne^{-.2075h}$

and the data for Table 5 is obtained from formulas (5) and (3).

Table 6 is the result of assuming $S = 1.5$ KTS, an increase in effectiveness obtained from some method of improving the technique of minehunting.

C. Casualties due to Combat.

Ref. (b) discusses the development by Lanchester of a set of differential equations that describe the number of casualties to be expected from a combat action between two forces. The equations as given there are,

$$\frac{dm}{dt} = -\frac{n}{1+E} G(t);$$

$$\frac{dn}{dt} = -\frac{mE}{1+E} G(t);$$

the solution is given as

$$n_o^2 - n^2 = (m_o^2 - m^2)E$$

E, the exchange rate, is here taken to be unity m_o and n_o are the number of Red and Blue forces, respectively, at the beginning of the combat and m and n are the number remaining at any time thereafter.

To apply the formula we have,

$m_o = 5000$

$n_o = 5000-50s-c$

$m = 4000$

n = number of Blue forces remaining when Red forces have suffered 1000 casualties

For the example started above we have

$$n_o = 5000-3500-178 = 3322$$

Then,

$$(5000)^2-(4000)^2 = (3322)^2-n^2$$

$$n = 1427$$

and

$$L = 3500-1427 = 2073$$

This quantity is tabulated for several values of mine combatting effectiveness in Tables 1-6 and plotted in Figs. 1 and 2.

PERSONNEL CASUALTIES vs No. of SWEEPERS

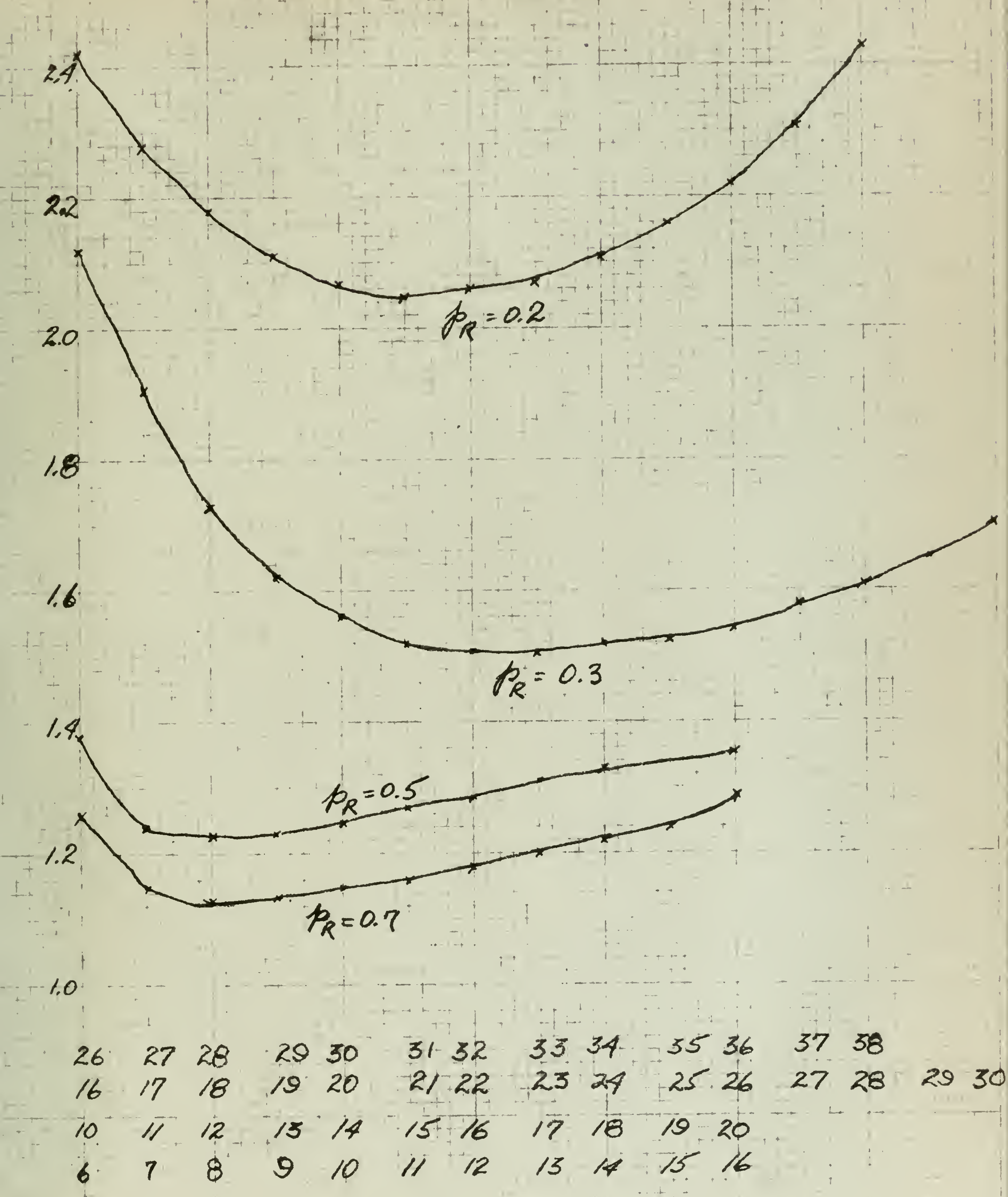

FIG 1

PERSONNEL CASUALTIES vs No. of HUNTERS

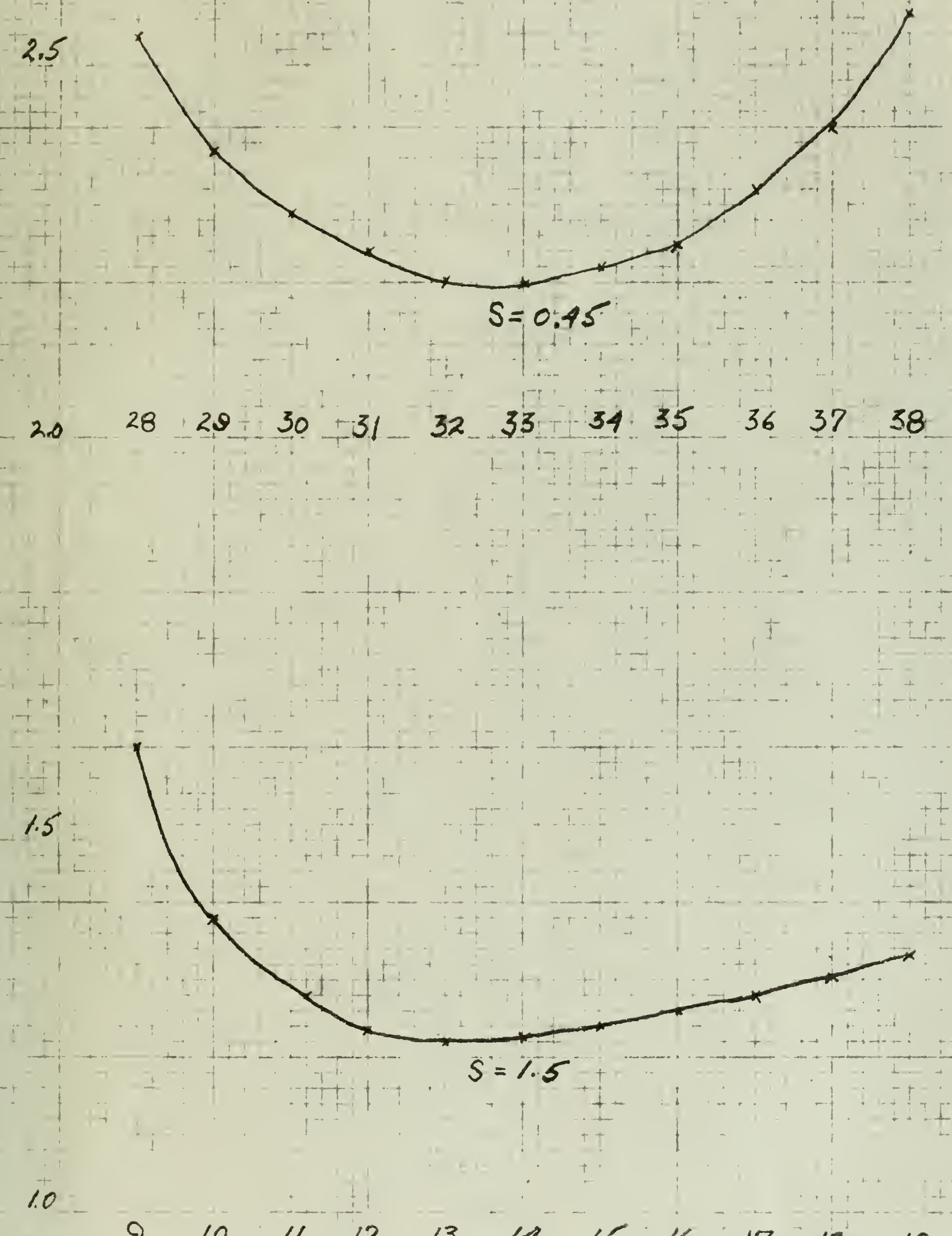

FIG 2

TABLE 1

(sweeping only)

$p_r = 0.20$

s	n'	p_c	C_m	n_o	m	L
25	3750	.147	551	3199	1111	2639
26	3700	.119	440	3260	1276	2424
27	3650	.097	354	3296	1365	2285
28	3600	.078	281	3319	1420	2180
29	3550	.063	224	3326	1436	2114
30	3500	.051	178	3322	1427	2073
31	3450	.041	141	3309	1396	2054
32	3400	.034	116	3284	1336	2064
33	3350	.027	90	3260	1276	2074
34	3300	.022	73	3227	1189	2111
35	3250	.018	59	3191	1087	2163
36	3200	.014	45	3155	977	2223
37	3150	.0113	36	3114	835	2315
38	3100	.00878	27	3073	666	2434

TABLE 2

(sweeping only)

$p_r = 0.3$

s	n'	p_c	C_m	n_o	n	L
16	4200	.132	554	3646	2072	2128
17	4150	.094	390	3760	2238	1912
18	4100	.067	275	3825	2373	1727
19	4050	.0472	191	3859	2427	1623
20	4000	.0332	133	3867	2440	1560
21	3950	.0234	92	3858	2426	1524
22	3900	.0165	64	3836	2391	1509
23	3850	.0115	44	3806	2342	1508
24	3800	.0082	31	3769	2282	1518
25	3750	.0057	21	3729	2229	1521
26	3700	.0040	15	3685	2161	1539
27	3650	.0029	11	3639	2074	1576
28	3600	.0021	8	3592	1985	1615
29	3550	.0014	5	3545	1897	1653
30	3500	.0008	3	3497	1802	1698

TABLE 3

(sweeping only)

$p_r = 0.5$

s	n'	p_c	C_m	n_o	n	L
10	4500	.039	176	4324	3114	1386
12	4400	.019	44	4356	3158	1242
13	4350	.0101	22	4328	3119	1231
14	4300	.0051	11	4289	3065	1235
15	4250	.0025	6	4244	3002	1248
16	4200	.0013	3	4197	2930	1270
17	4150	.0006	1	4149	2866	1284
18	4100	.00031	1	4099	2793	1307
19	4050	.00016	0	4050	2721	1329
20	4000	.00008	0	4000	2646	1354

TABLE 4

(sweeping only)

$p_r = 0.7$

s	n'	p_c	C_m	n_o	n	L
5	4750	.94	445	4305	3017	1733
6	4700	.029	137	4563	3438	1262
7	4650	.0088	41	4609	3499	1151
8	4600	.00265	12	4588	3471	1129
9	4550	.000797	4	4546	3416	1134
10	4500	.000240	1	4499	3353	1147
11	4450	.000072	0	4450	3291	1159
12	4400	.000002	0	4400	3219	1181
13	4350		0	4350	3150	1200
14	4300		0	4300	3081	1219
15	4250		0	4250	3013	1237
16	4200		0	4200	2913	1287

TABLE 5

(hunting only)

$p_r = 0.9 \quad S = 0.45$ KTS

h	n'	p_c	C_m	n_o	n	L
8	4600	.147	676	3924	2530	2070
9	4550	.076	345	4205	2947	1603
10	4500	.039	176	4324	3114	1386
11	4450	.0195	87	4363	3168	1282
12	4400	.0098	43	4357	3160	1240
13	4350	.0044	19	4331	3124	1226
14	4300	.0025	11	4289	3072	1228
15	4250	.00124	5	4245	3003	1247
16	4200	.00062	3	4197	2935	1265
17	4150	.00031	1	4149	2866	1284
18	4100	.000155	1	4099	2793	1307
19	4050	.000077	0	4050	2721	1329

TABLE 6

(hunting only)

$P_r = 0.9 \quad S = 1.5$ KTS

h	n'	P_c	C_m	n_o	n	L
27	3650	.139	508	3142	934	2716
28	3600	.114	412	3188	1079	2521
29	3550	.092	328	3222	1175	2375
30	3500	.076	267	3233	1205	2295
31	3450	.063	216	3234	1208	2242
32	3400	.051	174	3226	1186	2214
33	3350	.0415	139	3211	1145	2205
34	3300	.034	113	3187	1076	2224
35	3250	.028	91	3159	999	2251
36	3200	.023	73	3127	883	2317
37	3150	.018	58	3092	749	2401
38	3100	.015	47	3053	567	2533

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